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ORIGINAL ARTICLE

Metacognition and motivation in school-aged children with and without mathematical learning disabilities in Flanders

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Abstract
The role of metacognitive postdiction accuracy and autonomous and controlled motivation in mathematics was explored in elementary school children ($n = 208$) within two perspectives, related to sample characteristics. A first study was set up in a population-based cohort. A second study was set up with children with and without a documented mathematical disability. Both studies revealed a concurrent relation between the metacognitive postdiction skills of children and their mathematical accuracy and speed, leading to the practical recommendation that teachers should pay attention to the accuracy of self-judgments of children. In addition, controlled motivation was negatively related to the speed and accuracy in study 2. Children with mathematical learning disabilities (MLD) differed from peers without mathematical learning disabilities on postdiction accuracy and autonomous motivation. However, they did not differ significantly on controlled motivation, suggesting the importance of differentiating between controlled and autonomous motivation when analyzing motivation in mathematics education.

Keywords Calculation accuracy · Fact retrieval speed · Metacognitive postdiction accuracy · Self-judgment · Autonomous motivation · Controlled motivation · Mathematical learning disabilities

1 Introduction

Having good mathematical abilities is considered to be important (Jordan and Kaplan 2009; Jordan et al. 2010). Claessens and Engel (2013) revealed that pupils with high levels of performance in mathematics had a greater chance of later school success than pupils with low achievement levels in mathematics. Duncan and colleagues (2007, 2009) revealed that children who kept having low scores in mathematics during elementary school had 13% less chance of graduating from high school and 29% less chance of starting college education compared to typically developing peers.

Individual differences in mathematical abilities and disabilities can be seen as the outcome of a combination of predictors (Byrnes and Miller 2007; Geary 2011). In the research reported in this paper two of these predictors were studied, namely metacognition and motivation. Although there is plenty of evidence for metacognition and motivation as separate predictors of later mathematical achievement, there is little or inconsistent research simultaneously and empirically tapping the relationship between metacognition and motivation. In addition, the componential nature of mathematics is seldom taken into account. In this chapter, after a literature review, two studies are described, which study the relationship between mathematical accuracy and speed, leading to practical recommendations for mathematics educators of elementary school children with and without mathematical learning disabilities.

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2 Literature review

2.1 Mathematics within the opportunity–propensity model

Mathematical abilities refer to a componential construct (Dowker 2015) with at least two components. Ackerman and Ellingsen (2016) differentiated 'accuracy' from 'speed'. Mathematical problem solving requires 'accurate'

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procedures as well as fast retrieval of number facts or 'speed'. In addition, mathematical achievement can be studied within two perspectives. A first perspective is that there is a virtual continuum from very poor to very good mathematical problem solving. However, another perspective in the field is possible. Children with mathematics learning disabilities (MLD) are then considered as a specific and clinical group of children having a neurodevelopmental disorder, resulting in persisting difficulties with mathematical skills (< 10th percentile) despite the provision of interventions that target those difficulties. Most researchers currently report the prevalence of mathematical learning disabilities as ranging between 2 and 14% of children (Barbaresi et al. 2005; Dowker 2005).

Byrnes and Miller (2007) developed the Opportunity–Propensity framework, aiming to differentiate between opportunity and propensity predictors and to explain why these individual differences exist. Propensity predictors are variables that make people able (e.g., metacognition) and/or willing (e.g., motivation) to learn. Opportunity predictors include contexts and variables that expose children to learning content, such as classroom instruction (Byrnes and Miller 2007, 2016; Wang and Byrnes 2013). The opportunity–propensity model was tested using secondary datasets. In the first longitudinal study, researchers explained between 76.6% and 80.6% of the variance with this model in secondary school children in the United States. A path analysis confirmed the causality between opportunity and propensity predictors and mathematical achievement (Byrnes and Miller 2007). A second study with data from kindergarten up until primary school revealed additional evidence for the opportunity–propensity model with propensity predictors as the strongest predictors (Byrnes and Wasik 2009). Finally, Wang and colleagues (2013) found evidence for this model in lower-income pre-kindergarten children. Structural equation modelling confirmed the prediction by the opportunity–propensity model for early mathematical skills, with metacognition (or self-regulation) as one of the predictive propensity factors.

Since propensity factors were found to be the most strong predictors in younger children (Baten and Desoete 2018; Byrnes and Wasik 2009), this study focused on two specific propensity predictors, namely metacognition and motivation. Surprisingly few studies have been conducted to explore the combined effect of these predictors on mathematics in elementary school children. This study addressed this gap by investigating metacognition in addition to motivation as predictors of mathematical accuracy and speed in young children. It was studied if these two propensity predictors explain some of the variance in typical (study 1) and atypical (study 2) mathematical problem solving achievers. In the following paragraphs, research on metacognition and motivation related to mathematics is discussed.

2.2 Metacognition

Metacognition originates from cognitive information processing theory (Baten et al. 2017; Brown 1987; Flavel 1979; Schneider and Lockl 2002). The construct itself was introduced as the knowledge concerning one's own cognitive processes and products and anything related to them (for a review, see Schneider and Artelt 2010). There is consensus on the fact that metacognition plays a role in mathematical problem solving achievement (Özsoy and Ataman 2009; Schneider and Artelt 2010; Özsoy 2011; Morosanova et al. 2016), especially in challenging tasks, not overtaxing the capacity of children and in relatively new strategies that are being acquired (Carr et al. 1994; Carr and Biddlecomb 1998; Carr and Jessup 1995). During the initial stage of mathematical problem solving, when subjects build an appropriate representation of the problem, as well as in the final stage of interpretation and checking the outcome of the calculations, metacognition seems to be involved (Verschaffel 1999). Metacognition was found to prevent 'blind calculation' or a superficial 'number crunching' approach (e.g., answering '53' to the exercise'50 is 3 more than …', since 'more' is always translated into additions) in mathematics (Vermeer et al. 2000). Furthermore, metacognition allows students to use the acquired knowledge in a flexible, strategic way (Lucangeli et al. 1998).

Once metacognition gained popularity, most researchers agreed to differentiate a reflective component (or metacognitive knowledge) and an executive component (metacognitive skills). Metacognitive knowledge refers to the awareness of and reflection on cognitive strengths and weaknesses, the application of resources and strategies, and their situational appropriateness. Metacognitive knowledge consists of one's 'correct' and 'false' beliefs about the subject and nature of mathematics (Schneider and Artelt 2010). In mathematics, children may know, for example, that they have to check themselves in multi-digit divisions, but not while solving one-digit additions. Metacognitive skills encompass the 'active' control of engagement in learning, adapting to situational learning demands (Azevedo 2009). In mathematics, metacognitive skills refer to activities aimed at differentiating difficult exercises (e.g., 126:5 = ..) from the easy ones (e.g., 126 – 5 = ..), in order to be able to concentrate on and persist more in the high-effort tasks. In addition, metacognitive skills are involved in analyzing exercises (e.g., 'It is a division exercise in a number-problem format'), retrieving relevant domain-specific knowledge and skills (e.g., how to do divisions) and sequencing problem solving strategies (e.g., division of the hundreds, tens, units in mental mathematics). Metacognitive skills are also related to questions such as ‘am I following my plan?’, ‘is this plan working?’ ‘should I use paper and pencil to solve the division?’ and so on. Finally there is self-judging of the answer and of the process of getting to this answer (Desoete and Roeyers 2005). Metacognitive skills depend on procedural knowledge for the actual regulation of and control over one’s learning activities (Lucangeli et al. 1998; Desoete and Roeyers 2002; Wall et al. 2016).

There is some evidence for metacognitive knowledge as a necessary precursor to metacognitive skills (van der Stel and Veenman 2014). A combination of metacognitive knowledge and skill parameters explained 37% of the variance in mathematical problem solving in grade 2 and 3 (Desoete and Roeyers 2002). Metacognitive skills explained about 16% of the variance in mathematics. This combination was also successful to differentiate children with mathematical learning disabilities from below-average performing peers and average performers from expert problem solvers (Desoete et al. 2001). In addition, Schneider and Artelt (2010) revealed that the impact of metacognitive knowledge on mathematics performance was substantial, sharing about 15–20% of common variance in fifth grade (9- to 10-year-old children). Özsoy (2011) found an even stronger relation in fifth grade children, with 42% of the total variance of mathematics achievement explained by metacognitive knowledge and skills. Moreover, in 15-year-olds the Program for International Student Assessment (PISA) study demonstrated that roughly 18% of the variance in mathematics performance could be explained by the metacognition indicator (Schneider and Artelt 2010).

Despite all the emphasis on metacognition, several problems emerge in the assessment of metacognition, making study outcomes difficult to compare (Azvedo 2009, 2010; Desoete 2008). On the one hand, Veenman and his colleagues are sceptical and point to the lack of accuracy and the limited explained variance of learning outcomes of questionnaires (Veenman 2005, 2011; Veenman et al. 2006). On the other hand, think-aloud protocols were found to be very time-consuming techniques (Borkowski 1992; Veenman et al. 2006; Azevedo 2009; Schneider and Artelt 2010; Veenman 2005, 2011; Fleming and Lau 2014) and reviews indicated the value of less time-consuming self-judgments as worthy assessments of students’ achievement-related behaviors (Winne and Perry 2000).

The confidence of one’s performance after the task can be referred to as ‘postdiction’. Metacognitive postdictions revealed to be more accurate than metacognitive predictions (Hacker et al. 2000). Metacognitive postdiction accuracy improves with age (Desoete and Roeyers 2006) and task experience (Bol and Hacker 2012; Hacker et al. 2000). High achieving students were more accurate than their lower achieving peers (Hacker et al. 2000). Kruger and Dunning (1999, 2002) described this as a ‘double burden’ since poor performing students in this study did not only lack the necessary skills to estimate/calibrate successfully but also lacked the ability to recognize that their performance was poor.

2.3 Motivation

A problem with studies on metacognition is that they not only tap metacognitive predictors, but also address motivational variables (Borkowski and Thorpe 1994; Schneider and Artelt 2010). Motivation is an important propensity predictor of school achievement (Steinmayer and Spinath 2009). By reporting only on metacognition in studies addressing both metacognition and motivation, the importance and unique explained variance of metacognition might be overestimated. This study addresses this issue by exploring the combined effect of metacognition and motivation as propensity predictors, within the opportunity–propensity model, in order to gain more holistic insights on mathematics development. In what follows, we describe the conceptualization of motivation.

Self-determination theory claims that the more ‘autonomous’ (vs. controlled) the motivation is, the better (Chen et al. 2015; Vansteenkiste et al. 2009, 2014). Autonomy is defined as the psychological feeling of acting volitionally, without pressure from others (Van Petegem et al. 2015). The opposite, a more guilt-inducing style, can be described as controlling (Mageau and Vallerand 2003; Reeve 2009, 2016). Children who are ‘autonomously motivated’ study mathematics because of the personal relevance for a later academic career or for feelings of pleasure and passion.

Taylor and colleagues (2014) highlighted in a meta-analysis on 18 studies a positive relationship between autonomous motivation (where the force to fulfill a task is internal, e.g., passion) and general school achievement, in addition to a negative relationship between controlled motivation (where the force to fulfill a task is external, e.g., reward-related) and academic achievement. Motivation was found to predict the achievement in mathematics above general intelligence (Spinath et al. 2006, 2010). In the PISA study there were between 1 and 29% of explained variances attributable to motivational factors (Kriegbaum et al. 2015). In ninth graders, it was shown that perceived autonomy-support influenced students’ intrinsic motivation, resulting in better mathematics achievement (Froiland et al. 2016). A study on 114 school-aged children (grade 3 till 6) with and without MLD revealed no predictive value for autonomous motivation when investigated in combination with other predictors for mathematics. However the study demonstrated significant differences in autonomous motivation but not in controlled motivation between children with and without mathematical learning disabilities, after controlling for intelligence. Children with mathematical learning disabilities were less autonomously motivated than their peers (Baten and Desoete 2018).

2.4 Aims of the studies

Although there is plenty of evidence for metacognition and motivation as separate predictors of later mathematical achievement, there is little or inconsistent research simultaneously and empirically tapping the relationship between metacognition and motivation. In addition, since the current study takes into account the componential nature of mathematics, previous findings are extended. More specifically, the current study separately examined the prediction for two mathematical components, namely, procedural calculation 'accuracy' and fact retrieval fluency or 'speed' among children (Cohen et al. 2015; Pieters et al. 2015). Because mathematical achievement can be studied within two perspectives, two studies were set up. The first study was set up in a population-based cohort. In addition, study 2 was set up within the theoretical perspective of considering children with mathematical learning disabilities as a specific group of children with persistent difficulties and mathematical problem solving scores below critical cut-off scores. In the second study children with mathematical learning disabilities were compared with age-matched subjects in the same schools and in the same immediate mathematical environment (as opportunity factors). To conclude, the following major hypotheses were examined:

1. Postdiction accuracy (metacognition) and motivation will predict mathematics in both samples.
   - 1a In a population-based cohort, a positive relation is expected between metacognition (Özsoy 2011; Schneider and Artelt 2010) and autonomous motivation (Taylor et al. 2014) and a negative relation is expected between controlled motivation and mathematics performances (Taylor et al. 2014).
   - 1b In a sample of childeren with and without mathematical learning disabilities, metacognitive accuracy (Desoete et al. 2003; Lucangeli et al. 1998) and autonomous motivation (Baten and Desoete 2018) are expected to be positively related to mathematical abilities. No specific hypotheses are made for the different components (accuracy and speed) of mathematics.
2. Significant differences in metacognition (Lucangeli et al. 1998; Desoete et al. 2001) and motivation (Baten and Desoete 2018) are expected between children with and without learning disabilities. Children with mathematical learning disabilities are expected to be less autonomously motivated for mathematics and less accurate in their metacognitive postdictions compared to peers without learning disabilities.

3 Method

3.1 Design

Two cross-sectional studies were set up in the Dutch speaking part of Belgium. The first study was set up in a population-based cohort. In addition, study 2 was set up within the theoretical perspective of considering children with mathematical learning disabilities as a specific group of children with persistent difficulties and mathematical problem solving scores below critical cut-off scores. In the second study children with mathematical learning disabilities were compared with age-matched subjects in the same schools and in the same immediate mathematical environment (as opportunity factors).

3.2 Instruments

To measure the *speed* of fact retrieval, the Arithmetic Number Fact Test (de Vos 1992) was used. This test exists of five columns of exercises, one for each of the operations and one with a mix of the operations. Children had to solve as many additions (e.g., '7 + 2'), subtractions (e.g., '6 − 5'), multiplications (e.g., '5 × 8'), divisions (e.g.,'27:9′) or a mix of these exercises as possible in 5 min. The number of correct answers was used as outcome measure. This test was standardized for Flanders on a sample of 10,059 persons (Ghesquière and Ruijssenaars 1994). The psychometric value of the test was demonstrated with Cronbach's alpha of 0.90 (Desoete and Roeyers 2005).

To measure the *accuracy* or the procedural calculation skills of the child, the Cognitive Developmental skills in aRithmetic Test (Desoete and Roeyers 2002) was administered. In study 1 a selection of 20 exercises was used, whereas in study 2 the complete test (90 exercises) was solved by all participants. The test measures accuracy or proficiency to solve calculations in a number-problem or word-problem format (e.g., '283 times more than − 71 is …'; '27681:90 = …'; 'Wim has 4.8 kg of flour. Jan has a double amount of flour. How much flour do Jan and Wim have together?') without a time limit. The number of correct answers was calculated as outcome measure. The psychometric value of the full version was demonstrated with Cronbach's alpha of 0.89 (Desoete and Roeyers 2005). The short version correlated significantly ($r = 0.95$, $p < 0.001$) with the full version.

Metacognition was measured with a postdiction accuracy index computed by calculation of the absolute value of the difference between the score on mathematical accuracy and the self-estimation of the result by the participant. The nearer to zero, the more correct the postdiction was.

Motivation for mathematics was measured with the Dutch version of the Academic Self-Regulation Scale (Deci et al. 1989; Vansteenkiste et al. 2009) which consists of 24 questions which allow the calculation of the level of autonomous and controlled academic motivation. Children were asked questions such as 'I am motivated to study mathematics because …' in order to measure motivation with regards to mathematics specifically. The child had to respond on a 5-point rating scale to statements such as 'because I find this an important goal in my life' as an index of autonomous motivation and 'because other people (e.g., parents, friends, teachers) oblige me to do so' to measure controlled motivation. The score for each scale was calculated by averaging the scores on the items belonging to that scale. Cronbach's α was 0.85 for autonomous and 0.73 for controlled motivation.

3.3 Procedure

The participating children from study 1 were recruited in three randomly selected schools in Flanders. Study 2 was conducted with children with a documented history of mathematical learning disabilities (MLD) and peers without learning disabilities in the same schools. To keep the opportunity-factors as equal as possible between the two groups, children in the control group were recruited from the same classrooms as the children in the children with mathematical learning disabilities.

Participating in both studies was on a voluntary and anonymous basis and participation could be stopped at any time. Children's parents agreed to the research by signing informed consent forms. The studies were approved by the Ethical Committee of the Faculty of Psychology and Educational Sciences of Ghent University.

3.4 Participants

Study 1 was conducted in the general population with 63 children (40 females) between 9 and 12 years of age (mean age 10.76 years, $SD=1.13$). The mean visual spatial intelligence of the children was 110.83 ($SD=10.71$). Study 2 was conducted with 145 elementary school children (72 participants with and 73 participants without mathematical learning disabilities) in Flanders. The children with mathematical learning disabilities had a clinical diagnosis and mathematical abilities that were substantially and quantifiably below the performance level for the individual's chronological age, resistant to instruction (Ghesquière et al. 2014). The mean intelligence of the children was 98.96 ($SD=13.95$) on the Wechsler-Intelligence-Scale for Children-III (WISC-III-NL; Grégoire 2000; Kort et al. 2005; Wechsler 1991).

3.5 Statistical analyses

The correlations between the measures and the distribution of the estimation scores were calculated. To answer the first research question, two regression analyses were used. The first regression analysis was conducted on postdiction accuracy, autonomous motivation and controlled motivation predicting mathematical speed. The second one was conducted with the same propensity factors predicting mathematical accuracy in this population-based cohort (study 1: research question 1a). In addition two similar regressions were conducted to examine whether postdiction and motivation mattered in relation to mathematical speed and accuracy in children with and without a documented mathematical learning disability (study 2: research question 1b). Finally, to examine the differences between children with and without mathematical learning disabilities (research question 2), a multivariate analysis of variance (MANOVA) was conducted on metacognitive postdiction accuracy, autonomous motivation and controlled motivation.

4 Results

4.1 Research question 1a: study 1

Children in the general population estimated to have a total score of 16.05/20 ($SD=2.08$), whereas they had a real score of 14.76/20 ($SD=2.09$) on the abbreviated version of the mathematical accuracy test. In this sample 27% of the children overestimated their performance with 1 (12.7%), 2 (6.3%), 3 (6.3%) or 4 (1.6%) points. About 19% of the children had an accurate estimation of their mathematical accuracy. In addition, 54% of the children underestimated their performance with 1 (9.5%), 2 (16.7%), 3 (4.8%), 4 (7.9%), 5 (4.8%), 6 (4.8%), 7 (1.6%) or 8 (3.2%). The relation between the measures is described in Table 1.

There was no significant correlation between autonomous motivation and mathematical speed ($r=-0.17$, $p=0.194$) nor between autonomous motivation and mathematical accuracy ($r=-0.08$, $p=0.543$). In addition there was no significant correlation between controlled motivation and mathematical speed ($r=-0.22$, $p=0.085$) nor between controlled motivation and mathematical accuracy ($r=-0.11$, $p=0.378$).

The regression of motivation predicting 0.6% of mathematical speed was not significant [F (2, 59) = 1.87; $p=0.164$]. Only when metacognition (postdiction) was included, the prediction of 15% of mathematical speed became significant [F (3,59) = 3.55; $p=0.020$], see Table 2.

Metacognition (postdiction) had an added value in the prediction of mathematical speed. The data in Table 2 revealed that a more accurate estimation of their own

Table 1 Correlation between the measures in a general population

	Accuracy	Autonomous motivation	Controlled motivation	Postdiction
Mathematical speed	0.53**	− 0.17	− 0.22	− 0.35**
Mathematical accuracy		− 0.08	− 0.11	− 0.65**
Autonomous motivation			0.28*	0.21
Controlled motivation				0.17

*$p \leq 0.05$

**$p \leq 0.01$

Table 2 Prediction of mathematical speed in a general population

	B	ß	*t*	*p*
Constant	153.04		9.59	0.000
Autonomous motivation	− 1.59	− 0.06	− 0.46	0.645
Controlled motivation	− 5.24	− 0.15	− 1.16	0.252
Metacognition	− 3.92	− 0.32	− 2.56	0.013**

**$p \leq 0.01$

Table 3 Prediction of the mathematical accuracy in a general population

	B	ß	*t*	*P*
Constant	14.97		11.82	0.000
Autonomous motivation	0.17	0.07	0.63	0.528
Controlled motivation	− 0.04	− 0.01	− 0.11	0.909
Metacognition	− 0.82	− 0.68	− 6.73	0.000**

**$p \leq 0.01$

results and less deviation from the correct result (postdiction) was related to better mathematical speed in the general population.

The second regression of motivation predicting 0.01% of *mathematical accuracy* was not significant [F (2, 59) = 0.42; $p = 0.656$]. Only when metacognition (postdiction) was included the regression became significant [F (3,59) = 15.61; $p < 0.001$] with an explained variance of 45.5%, see Table 3.

Metacognition predicted mathematical accuracy in general population.

4.2 Research question 1b: study 2

The lack of significant correlation between mathematics and intelligence might be due to the sampling (small sample out of the general population). Therefore study 2 was set up for children with and without a documented history of mathematical learning disablities.

Children without MLD in this study estimated to have a mathematical accuracy score of 75.51/90 (SD = 10.83), whereas they had a real score of 77.78/90 (SD = 7.62). In this sample 56.2% of the children overestimated their performance with more than 40 points (1.4%), between 31 and 40 points (3.5%), between 21 and 30 points (4.2%), between 11 and 20 points (7.7%), between 6 and 10 points (13.9%), or between 1 and 5 points (19.3%) points. About 3.4% of the children had an exact and accurate estimation of their mathematical accuracy. About 17.9% underestimated their accuracy between 1 and 5 points. Some children underestimated their mathematical accuracy between 6 and 10 points (12.4%), between 11 and 20 points (11.2%), between 21 and 30 points (4.2%) or with more than 40 points (1.4%).

Children with M LD estimated to have a total mathematical accuracy score of 64.13/90 (SD = 17.76), whereas they had a real score of 63.18/90 (SD = 11.17). In this sample 43.1% of the children overestimated their performance. The distribution of the overestimation varied, with an overestimation of more than 40 points (2.8%), between 31 and 40 points (2.8%), between 21 and 30 points (8.4%), between 11 and 20 points (7.0%), between 6 and 10 points (11.3%) to a rather limited overestimation between 1 and 5 points (11.2%) points. About 2.8% of the children with mathematical learning disabilities had an exact and accurate estimation of their mathematical accuracy. About 15.4% of the children with mathematical learning disabilities had a rather limited underestimation of between 1 and 5 points. Other children with mathematical learning disabilities underestimated their mathematical accuracy between 6 and 10 points (15.3%), between 11 and 20 points (15.4%), between 21 and 30 points (8.4) or with more than 40 points (2.8%).

For the correlations between the measures, see Table 4.

There was a significant correlation between autonomous motivation and mathematical speed (r = 0.16, $p = 0.034$) and between autonomous motivation and mathematical accuracy (r = 0.18, $p = 0.017$). There was a significant correlation between controlled motivation and mathematical speed (r = − 0.19, $p = 0.013$) and between controlled motivation and mathematical accuracy (r = − 0.26, $p = 0.001$).

The regression of motivation predicting 0.6% of the variance of *mathematical speed* was significant [F (2, 169) = 5.67; $p = 0.004$]. Especially controlled motivation mattered. When metacognition (postdiction) was added

Table 4 Correlation between the measures in children with and without learning disabilities

	Accuracy	Autonomous motivation	Controlled motivation	Metacognition
Mathematical speed	0.65**	0.16*	−0.19*	−0.32**
Mathematical accuracy		0.18*	−0.26**	−0.26**
Autonomous motivation			0.01	−0.06
Controlled motivation				−0.19*

*$p \leq 0.05$

**$p \leq 0.01$

Table 5 Prediction of mathematical speed in children with and without disabilities

	B	ß	*T*	*p*
Constant	111.08		11.01	0.000
Autonomous motivation	3.51	0.12	1.59	0.113
Controlled motivation	−6.16	−0.19	−2.38	0.018*
Metacognition	−068	−0.27	−3.41	0.001**

*$p \leq 0.05$

**$p \leq 0.01$

Table 6 Prediction of mathematical accuracy in children with and without disabilities

	B	ß	*t*	*P*
Constant	80.52		19.39	0.000
Autonomous motivation	1.27	0.10	1.39	0.166
Controlled motivation	−3.09	−0.21	−2.92	0.004**
Metacognition	−0.49	−0.43	−5.93	0.000**

*$p \leq 0.05$

**$p \leq 0.01$

14.9% of the variance could be predicted [F (3,142) = 8.08; $p < 0.001$], see Table 5.

Metacognition (postdiction) had an added value in the prediction of mathematical speed of 14.3% (the prediction changed from 0.6 to 14.9%). A smaller difference between the real and self-estimated score was related to better fact retrieval skills (or mathematical speed) in children with and without mathematical learning disabilities.

The regression of motivation predicting 10.2% of *mathematical accuracy* was significant [F (2, 171) = 9.55; $p < 0.001$]. When metacognition (postdiction) was included [F (3, 144) = 18.29; $p < 0.001$] there was a significant prediction of 28% of the variance, see Table 6.

Metacognition (postdiction) had an added value in the prediction of mathematical accuracy of 17.8% (the prediction changed from 10.2 to 28%). A smaller difference between the real and self-estimated score was related to better procedural calculation skills (or mathematical accuracy) in children with and without mathematical learning disabilities.

Table 7 Motivation and metacognition in children with and without disabilities

	Children with mathematical disabilities M (SD)	Peers without disabilities M (SD)	*F (3, 141)*
Autonomous motivation	2.85 (0.89)	3.38 (0.93)	12.27*
Controlled motivation	2.93 (0.75)	2.72 (0.88)	2.31
Metacognitive postdiction	13.91 (12.23)	7.13 (7.22)	16.57**

**$p \leq 0.01$

Controlled motivation and metacognition (postdiction) were significant predictors for mathematical accuracy in children with and without mathematical learning disabilities.

4.3 Research question 2: study 2

To answer the second research question, the motivation and metacognition postdiction results of children with and without mathematical learning disabilities were compared. Both groups differed on motivation and on metacognition (postdiction) on the multivariate level [F (3, 141) = 10.23; $p < 0.001$; $\eta_p^2 = 0.18$]. On the univariate level there were significant differences for autonomous motivation [F (1, 143) = 12.28; $p < 0.001$; $\eta_p^2 = 0.08$] and postdiction [F (1, 143) = 16.57; $p < 0.001$; $\eta_p^2 = 0.10$] but not for controlled motivation [F (1, 143) = 2.31; $p = 0.131$; $\eta_p^2 = 0.02$]. For M and SD, see Table 7.

Both groups differed on autonomous motivation and on metacognitive postdiction.

Children with mathematical learning disabilities were less autonomously motivated and their postdiction scores differed more from their real mathematical accuracy scores than did those of peers without disabilities.

5 Summary of results

To summarize, motivation did not predict mathematic in the general population (study 1), but a more accurate estimation of the own results and less deviation from the correct result was related to better mathematical speed. The combination of motivation and metacognition predicted 15% of the variance in mathematical speed and about 45% of the variance in mathematical accuracy. In addition, in the sample of children with and without a documented history of mathematical learning disabilities (study 2) there was a significant relationship between autonomous motivation and mathematical accuracy, and controlled motivation mattered for mathematical speed. Metacognition predicted about 14% of the variance in mathematical speed and about 18% of the variance in mathematical accuracy. Controlled motivation predicted about 10% of the variance in mathematical accuracy. A smaller difference between the real and self-estimated score was related to better fact retrieval and procedural calculation skills. In addition, children with mathematical learning disabilities were less autonomously motivated and their metacognitive postdiction scores differed more from their real mathematical accuracy scores than did those of peers without disabilities.

6 Discussion and conclusion

Trying to understand the nature of mathematical cognition has been a subject of research for many years. A great number of factors have been recognized as important for the development of mathematical performance (Dowker 2015; Geary 2011). Although these variables explain a part of the variance in mathematical ability, the nature and the concordance (i.e., covariation) between the different predictors remain poorly understood. Therefore, the Opportunity–Propensity Framework (Byrnes and Miller 2007; Wang et al. 2013) was proposed. Two studies were set up to study the relation between two propensity factors and mathematics, namely metacognition (assessed with postdiction scores) and motivation.

When evaluating the value of *metacognition as propensity factor*, in line with Hacker and colleagues (2000), Lin and colleagues. (2001) and Kruger and Dunning (1999, 2002), in study 1 about one-fourth of the children in the general population overestimated their mathematical performance on a short test. In study 2 more than half of the children without learning disabilities and four out of ten children with mathematical learning disabilities overestimated their performance on the longer test. However several children were accurate or underestimated their mathematical performances, showing much variation in the distribution of over- and underestimation. Additional research is needed to understand why there is so much difference in how accurate children are in estimating their performance. Nevertheless postdictions explained some of the variance in mathematical speed and accuracy in both studies. A more accurate estimation (postdiction) of their own results resulting in less deviation from the correct result was related to better mathematical speed and accuracy.

In line with Taylor and colleagues (2014), in study 2 there were significant positive correlations between motivation, mathematical speed and mathematical accuracy. In line with the Self Determination theory (Ryan and Deci 2002), there were negative significant correlations between controlled motivation and mathematics. Less external or reward-dependent (controlled) motivation and more accurate self-judgments (metacognitive postdictions) were significant predictors of mathematical speed and accuracy.

Finally, answering the second research question, in line with Baten and Desoete (2018), children with and without mathematical learning disabilities, matched on school opportunities, differed on autonomous motivation (where the force to fulfill a task is internal, e.g., passion), but not on controlled motivation (where the force to fulfill a task is external; e.g., a reward). In addition children with mathematical learning disabilities were as a group less accurate in their self-judgements, compared to their peers without learning disabilities who received the same instruction and opportunities at school. Since metacognition might not develop automatically in all children (Desoete et al. 2003), these findings seem to lead to the practical recommendation that teachers should pay attention to the autonomous motivation and accuracy of self-judgments in mathematics education.

There are certainly some limitations in these studies. The first limitation is the fact that although the metacognitive concept is more than 40 years old, researchers keep using different concepts (Tarricone 2011) and tests (Desoete 2008). Future research with other metacognitive tasks is needed to study the impact of the operationalization of metacognition. In this study the choice for 'postdiction' represented a restricted assessment of metacognition, not including the metacognitive skills employed during task performance. Other measures (Borkowski 1992; Fleming and Lau 2014) such as think aloud protocols, might result in different findings (Fleming and Lau 2014). In addition the 'accuracy' of postdictions was computed, in line with Desoete and Roeyers (2006) based on the comparison of the postdiction and the real performance. The nearer to zero, the more correct the postdiction was. However, there are other calculation procedures and indices possible (Boekaerts and Rozendaal 2010; Desender et al. 2017; Koriat 2007), such as the ones used in calibration accuracy and metacognitive monitoring studies to identify correct and/or incorrect performances (Chen 2002; Schraw et al. 2013, 2014; García et al. 2016). This aspect might have resulted in different study outcomes. In addition, the sample size in study 1 was rather small. Obviously sample size is not a problem for significant differences. However, when analyses have insufficient power and were not significant (e.g., for autonomous motivation and mathematics in study 1), a risk of type 2- or β-mistakes (concluding from the cohort that there were no differences although in reality there were differences in the population) cannot be excluded. Additional research with larger groups of participants is indicated. Finally, the relationships within the subject of mathematics might vary across years, indicating that our results might not be extrapolated to younger or older students and other context or opportunity variables, such as home environment, which should be included in order to obtain a complete overview of the development of these children. Such studies are currently being conducted.

Nevertheless, both studies highlighted much variation in the distribution of over-and underestimation of mathematical performances, with self-judgment (metacognitive accuracy) positively related to mathematical speed and accuracy. In addition, controlled motivation (or the feeling of pressure from others) was negatively related to mathematical speed and accuracy. Moreover, children with mathematical learning disabilities were less autonomously motivated, but there was no significant difference with peers on controlled motivation, suggesting the importance of differentiating between controlled and autonomous motivation when analyzing motivation in mathematics education. In conclusion, the present results confirm the role of metacognition on top of motivation as propensity predictor in elementary school mathematics. Since metacognition is teachable (Desoete et al. 2003; Baten et al. 2017), these studies suggest that instructional designs including feedback on the accuracy of self-judgments might be indicated.

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